in this manner indicate the existence of a libration. According to all indications, the motion of these planets is therefore entirely stable. The very extensive and accurate computations of Berberich have furnished very reliable sets of osculating elements of Andromache. With Berberich's elements osculating for 1877 and Miss Levy's development of the perturbations on the basis of the revised tables, an observation for the year 1920 has been represented with a remarkable degree of accuracy, so that in this case, also, the process of computing the special perturbations from opposition to opposition and frequent observations may be abandoned for many decades to come.

It is hoped that the results for 10 Hygiea and 175 Andromache here presented will prove a determining factor in the methods hereafter to be applied by astronomers in deriving the approximate perturbations by Jupiter of planets of the Hecuba Group and of other groups with mean motion commensurable to that of Jupiter to which Bohlin's original group theory is applicable with proper modifications and extensions. These studies should form a most fruitful field of research in theoretical astronomy.

A METHOD OF DERIVING THE DISTANCE OF THE A-TYPE STARS

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Our knowledge of the distances of individual stars of the A-type of spectrum is obtained almost wholly from three sources.

1. Trigonometric parallaxes.

2. Dynamical, sometimes called hypothetical, parallaxes of binary stars.

3. Parallaxes derived from group motion.

The third method probably furnishes the most accurate values for all such stars as belong to well-recognized groups. In the case of the Taurus Group, for example, the distances are known with a high degree of precision. The trigonometric parallaxes are, of course, most valuable in the case of large values for which the probable error bears a relatively small ratio to the quantities measured. The dynamical parallaxes derived from binary stars with well-determined orbital motion are of excellent quality, but are affected to some extent by the uncertainty in the value of the mass-factor for the A-type stars which enters directly into the computation.

A list of 100 stars with distances and absolute magnitudes obtained by these methods has formed the basis of an attempt upon our part to investigate a possible correlation between absolute magnitude and the intensities of certain spectral lines similar to that which has been used successfully in the case of stars of the later types of spectrum. The material used includes the stars for which spectra have been obtained with the Cassegrain spectrograph at Mount Wilson, and the values of the parallaxes and absolute magnitudes have been taken from the lists of trigonometric parallaxes of various observers, an unpublished investigation by Russell on dynamical parallaxes, and the memoir of Rasmuson on moving clusters. Corrections have been applied to the dynamical parallaxes to reduce them to the values corresponding to the most probable value of the mass of the A-type stars.

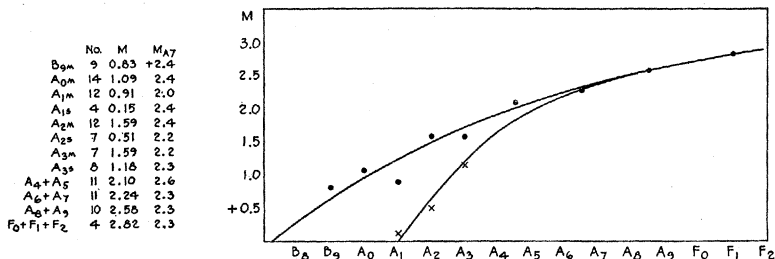|	No.	M	M_{A7}
B_{9m}	9	0.83	+2.4
A_{0m}	14	1.09	2.4
A_{1m}	12	0.91	2.0
A_{1s}	4	0.15	2.4
A_{2m}	12	1.59	2.4
A_{2s}	7	0.51	2.2
A_{3m}	7	1.59	2.2
A_{3s}	6	1.18	2.3
A_4+A_5	11	2.10	2.6
A_6+A_7	11	2.24	2.3
A_8+A_9	10	2.58	2.3
$F_0+F_1+F_2$	4	2.82	2.3

FIG. 1

Curves showing the relationship between absolute magnitude and spectral type for stars with parallaxes determined trigonometrically, dynamically and from group motions. Crosses refer to spectra with sharp lines A_1s, points to spectra with diffuse or nebulous lines as A_1n. In the present case this distinction is not carried into the more advanced sub-divisions,. M_{A7} denotes the absolute magnitudes reduced to spectral type A_7 by means of these curves.

The starting point of the investigation and, as it proved, its most important feature, was an accurate determination of spectral type and the characteristics of the spectral lines. The stars were classified according to the Harvard scale as A_0, A_1, A_2, etc., and the spectra were designated n or s according as the lines are nebulous or sharp. This distinction is marked among the stars of the earlier subdivisions but becomes less certain at $A5$ or later. In addition to these two classes with sharp or nebulous lines is the small group of stars, of which α Cygni is the most prominent example, which are characterized by the Harvard observers as c-stars. These have exceptionally sharp and narrow lines and the enhanced lines as a rule are remarkably strong. In most of them the enhanced lines of strontium at λ4077 and λ4215 are very intense.

The following table gives the absolute magnitudes M of the stars arranged according to spectral type.

	NO.	M	MA7		NO.	M	MA7		NO.	M	MA7
B9n	9	+0.83	+2.4	A2n	12	+1.59	+2.4	A4+A5	11	+2.10	2.6
A0n	14	1.09	2.4	A2s	7	0.51	2.2	A6+A7	11	2.24	2.3
A1n	12	0.91	2.0	A3n	7	1.59	2.2	A8+A9	10	2.58	2.3
A1s	4	0.15	2.4	A3s	8	1.18	2.3	$F_0+F_1+F_2$ 4		2.82	2.3

An inspection of these results shows a steady increase in the values of the absolute magnitude with advancing spectral type and a marked difference in brightness between the stars of the same type according as the lines are diffuse or sharp. At type $A4$ and later no distinction is made as regard this latter characteristic except in the case of c-stars.

If these results are plotted and curves drawn through them we can from the curves read off the average absolute magnitude for each spectral type. These are as follows:

	DIFFUSE	SHARP		DIFFUSE	ARP
B8	+0.4		A5	+2.1	+1.8
B9	0.7		A6	2.2	2.1
A0	1.0		A7		2.3
A1	1.2	0.0	A8		2.5
A2	1.5	0.6	A9		2.6
A3	1.7	1.2	F0		2.7
A4	1.9	1.5	F1		2.8
			F2		2.9

Finally if the absolute magnitudes of the original groups of stars are all reduced to the same spectral type $A7$ by means of these curves we obtain the results given under MA_7 in the first table. These show a remarkable degree of accordance, the total range among the groups being from 2.0 to 2.6 with an average deviation from the mean of 0.1 magnitude.

Within the groups themselves the probable error for an individual star is given below. This assumes that there is no dispersion in absolute magnitude around the mean value for a group of stars of a given spectral type with definite characteristics to their lines. That the observational results indicate this dispersion to be small is shown by the moderate size of the probable errors in spite of the fact that these are in some cases probably increased greatly by errors in the trigonometric parallaxes.

	NO.			NO.	
A9n	9	±0.48	A3n	7	±0.54
A0n	14	0.46	A3s	8	0.07
A1n	12	0.75	A4+A5	11	0.49
A1s	4	0.16	A6+A7	11	0.69
A2n	12	1.02	A8+A9	10	1.24
A2s	7	0.67	F0+F1+F2	4	0.91

The result of the comparison is to indicate that for normal A-type stars the absolute magnitude and parallax can be determined with a precision quite comparable with other methods from an accurate observation of spectral type and the characteristics of the spectral lines. For the c-stars, at least in such cases as α Cygni, this is not the case, but these stars appear to show in their spectra the phenomena connected with abnormal intrinsic brightness, and like the Cepheid variables of the F-type which they resemble noticeably there is every reason to believe they require a separate method of reduction.

Of the application of these results to individual stars it is possible to refer only very briefly but the following results may be of interest.

	TYPE	COMP. π	OBS. π
α Lyrae	A1s	0.09	0.10
ζ^1 Ursae Majoris	A2s	0.044	0.045
Taurus Group (26 stars)	A0–F	0.024	0.023
Praesepe (4 stars)	A2–A6	0.011	0.011

The close relationship of these results to certain observations by other investigators is of interest. It was found by Hertzsprung and later confirmed by Kohlschütter[2] that the stars in the Taurus cluster show approximately a linear relationship between color index or spectral type and absolute magnitude. These results deal in general with the larger spectral classes and later types and not with the subdivisions of a single type as is the case here.

In a recent investigation of the luminosity of the A-type stars, Lindblad[3] found that the intensity of the continuous spectrum in a region between λ3895 and λ3907 was less for the intrinsically fainter stars of this type than for the brighter stars. This effect he ascribes to the increased intensity of the wings of $H\zeta$ and the greater width and intensity of some of the other lines, especially those of iron and silicon, in this region. In other words the spectra of the fainter stars are diffuse as compared with those of the brighter stars, a result in agreement to that found in this investigation.

There seems to be little doubt that great intrinsic luminosity is associated with sharp and narrow lines in all spectral types and that this is due to the low density in the atmospheres of such stars. In stars of various spectral types, such as β Orionis, α Cygni, the Cepheid variables and α Orionis this effect is very marked, and the change in the character of the lines of the variable star o Ceti from maximum when they are well-defined to minimum when they are diffuse and vague is an excellent illustration of the same phenomenon.

[1] Hertzsprung, *Astron. Nachr.*, **209** (115).
[2] Kohlschütter, *Ibid.*, **211** (289).
[3] Lindblad, *Astroph. J.*, **55** (85).